§7.1 Integration by Parts (IBP)

IBP Formula(s): f-g version: $$\int f(x)\cdot \underbrace{g'(x)dx} = f(x)\cdot g(x) - \int g(x)\cdot \underbrace{f'(x)dx}$$

u-v version:
$u=f(x)$, $du=f'(x)dx$.
$v=g(x)$, $dv=g'(x)dx$

$$\boxed{\int u\cdot dv = u\cdot v - \int v\cdot du}$$

Definite Integral Version: $$\int_a^b u\cdot dv = u\cdot v\Big|_a^b - \int_a^b v\cdot du$$

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eg. 1. Evaluate $\int \underbrace{x}_{u}\cdot \underbrace{e^x dx}_{dv}$.

$$\boxed{\begin{array}{l}\text{IBP Setting: } u=x, \quad du=1\cdot dx \\ \qquad dv=e^x dx, \quad v=e^x\end{array}}$$

$$\overset{IBP}{=} u\cdot v - \int v\cdot du = x\cdot e^x - \int e^x dx = \boxed{x\cdot e^x - e^x + C}$$

eg. 2 (Definite Version) $\int_1^{e^2} \underbrace{\ln x}_{u}\cdot \underbrace{dx}_{dv}$.

$$\boxed{\begin{array}{l}\text{IBP Setting: } u=\ln x, \quad dv=dx \\ \qquad du=\frac{1}{x}dx, \quad v=x.\end{array}}$$

$$= u\cdot v - \int v\cdot du$$
$$= \ln x\cdot x\Big|_1^{e^2} - \int_1^{e^2} x\cdot\frac{1}{x}dx = \ln x\cdot x\Big|_1^{e^2} - \int_1^{e^2} 1\cdot dx$$
$$= \ln x\cdot x\Big|_1^{e^2} - x\Big|_1^{e^2}$$

Hint: $\ln e^2 = 2\ln e = 2$, $\ln 1 = 0$

$$= \ln e^2\cdot e^2 - \ln 1\cdot 1 - (e^2-1)$$
$$= 2e^2 - e^2 + 1 = \boxed{e^2+1}$$

Motivation: (FTC) $\int [f(x)\cdot g(x)]'\,dx = f(x)\cdot g(x) + C$

Product Rule: $\int [f(x)\cdot g'(x) + f'(x)g(x)]\,dx = f(x)\cdot g(x) + C$. (alternating form)

$$\Leftrightarrow \int f(x)\cdot g'(x)\,dx + \int f'(x)\cdot g(x)\cdot dx = f(x)\cdot g(x) + C$$

Remark: ① $u=f(x)$ should be the part with simpler derivative.
$dv=g'(x)dx$ should be the part with simpler anti-derivative.

② The key step is to integrate $g'(x)$ to find $g(x)=v=\int g'(x)dx$. from $dv=g'dx$

- Typical IBP problem (Typical choice for u and dv).

① $\int \underbrace{\text{Polynomial}}_{u} \times \underbrace{\sin/\cos/\exp\, dx}_{dv}$ ② $\int \underbrace{\ln x/\tan^{-1}x/\sin^{-1}x}_{u} \times \underbrace{\square\, dx}_{dv}$.

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eg.3. $\int \underbrace{(t^2+2t)}_{u} \cdot \underbrace{e^{2t}\cdot dt}_{dv}$ 1st IBP: $u=t^2+2t$, $du=(2t+2)\,dt$

$\boxed{dv=e^{2t}dt \Rightarrow v=\int e^{2t}dt=\frac{1}{2}e^{2t}}$

$\overset{\text{1st IBP}}{=} (t^2+2t)\cdot\frac{1}{2}e^{2t} - \int \frac{1}{2}\cdot e^{2t}\cdot(2t+2)\cdot dt$

$= (t^2+2t)\cdot\frac{1}{2}e^{2t} - \int \underbrace{e^{2t}}_{dv}\cdot\underbrace{(t+1)}_{u}\cdot dt$ 2nd IBP: $u=t+1$, $du=1\cdot dt$

$dv=e^{2t}dt$, $v=\frac{1}{2}e^{2t}$

$\overset{\text{2nd IBP}}{=} (t^2+2t)\cdot\frac{1}{2}e^{2t} - \left[(t+1)\cdot\frac{1}{2}e^{2t} - \int\frac{1}{2}e^{2t}\cdot dt\right]$

$= (t^2+2t)\cdot\frac{1}{2}e^{2t} - (t+1)\cdot\frac{1}{2}e^{2t} + \int\frac{1}{2}e^{2t}dt$ (Evaluate the last integral)

$= \boxed{(t^2+2t)\cdot\frac{1}{2}e^{2t} - (t+1)\frac{1}{2}e^{2t} + \frac{1}{2}\cdot\frac{1}{2}e^{2t} + C} = \boxed{\left(\frac{t^2}{2}+\frac{t}{2}-\frac{1}{4}\right)e^{2t}+C}$

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eg.4. $\int \underbrace{\tan^{-1}x}_{u}\cdot\underbrace{2x\,dx}_{dv}$ IBP: $u=\tan^{-1}x$, $du=\frac{1}{1+x^2}dx$.

$dv=2x\,dx$, $v=\int 2x\cdot dx = x^2$

$= \tan^{-1}x\cdot x^2 - \int\frac{1}{1+x^2}\cdot x^2\cdot dx$. Hint: $\int\frac{x^2}{1+x^2}dx = \int\frac{x^2+1-1}{1+x^2}dx = \int\frac{x^2+1}{1+x^2}-\frac{1}{1+x^2}\,dx$

$= \int 1-\frac{1}{1+x^2}\,dx$

$= x-\tan^{-1}x+C$

$= \tan^{-1}x\cdot x^2 - (x-\tan^{-1}x)+C$

$= \boxed{\tan^{-1}x\cdot x^2 - x + \tan^{-1}x + C}$

- Three useful formulas for dv and v.

① If $dv=\sin(ax+b)\cdot dx$, then $v=\int\sin(ax+b)\cdot dx = \frac{-\cos(ax+b)}{a}$.

② If $dv=\cos(ax+b)\cdot dx$, then $v=\int\cos(ax+b)\,dx = \frac{\sin(ax+b)}{a}$

③ If $dv=e^{ax+b}dx$. then $v=\int e^{ax+b}dx = \frac{e^{ax+b}}{a}$

Loop Trick (Appendix 1)

Evaluate $\int \underbrace{e^{2x}}_{u} \cdot \underbrace{\sin 3x \cdot dx}_{dv}$.

Solution: (IBP twice, set up AN EQUATION for the INTEGRAL, then solve for it)

$$\boxed{\begin{array}{ll} u = e^{2x}, & dv = \sin 3x \cdot dx \\ \Downarrow & \Downarrow \\ du = 2e^{2x}dx, & v = -\frac{1}{3}\cos 3x \end{array}}$$

$$\int e^{2x} \cdot \sin 3x \cdot dx = \int u\,dv = u \cdot v - \int v\,du$$

$$= e^{2x} \cdot (-\tfrac{1}{3}\cos 3x) - \int \underbrace{(-\tfrac{1}{3}\cos 3x)}_{dv} \cdot \underbrace{2e^{2x}}_{u} dx.$$

$$\boxed{\begin{array}{ll} u = 2 \cdot e^{2x} & dv = -\frac{1}{3}\cos 3x\,dx \\ \Downarrow & \\ du = 4 \cdot e^{2x}, & v = (-\frac{1}{3}) \cdot \frac{1}{3}\sin 3x = -\frac{1}{9} \cdot \sin 3x. \end{array}}$$

$$\int e^{2x} \cdot \sin 3x \cdot dx = e^{2x} \cdot (-\tfrac{1}{3}\cos 3x) - \int \underbrace{2e^{2x} \cdot (-\tfrac{1}{3}\cos 3x)\,dx}$$

$$= e^{2x} \cdot (-\tfrac{1}{3}\cos 3x) - \left[2 \cdot e^{2x} \cdot (-\tfrac{1}{9}\sin 3x) - \int (-\tfrac{1}{9}\sin 3x) \cdot 4 \cdot e^{2x} dx\right]$$

Simplify: $\boxed{\int e^{2x} \cdot \sin 3x\,dx} = e^{2x} \cdot (-\tfrac{1}{3}\cos 3x) + \frac{2}{9} \cdot e^{2x} \cdot \sin 3x - \frac{4}{9}\boxed{\int \sin 3x \cdot e^{2x} dx}$.

Notice the expressions in the two boxes are the same, solve for the BOX

$$\int e^{2x} \cdot \sin 3x \cdot dx + \frac{4}{9}\int \sin 3x \cdot e^{2x} dx = -\frac{1}{3}e^{2x} \cdot \cos 3x + \frac{2}{9} \cdot e^{2x} \cdot \sin 3x.$$

$$(1 + \tfrac{4}{9}) \cdot \int e^{2x} \sin 3x \cdot dx = -\frac{1}{3}e^{2x} \cdot \cos 3x + \frac{2}{9}e^{2x} \cdot \sin 3x$$

$$\frac{13}{9} \cdot \int e^{2x} \sin 3x\,dx = -\frac{1}{3}e^{2x} \cdot \cos 3x + \frac{2}{9}e^{2x} \cdot \sin 3x$$

$$\int e^{2x} \cdot \sin 3x\,dx = \frac{9}{13}\left[-\frac{1}{3} \cdot e^{2x} \cdot \cos 3x + \frac{2}{9} \cdot e^{2x} \cdot \sin 3x\right] + C$$

§7.2 Trigonometric Integrals

● ① sin-cos product with one ODD power. $\boxed{\text{Rule: Find the "ODD part", substitute THE OTHER one}}$

★ eg 1. $\int \cos^3 x \cdot \sin^{100} x\, dx$, Hint: cos has power 3 (odd), then substitute sin (the other one)

$u = \sin x$, $du = \cos x \cdot dx \Rightarrow \boxed{\frac{du}{\cos x} = dx}$

$= \int \cos^3 x \cdot u^{100} \cdot \frac{du}{\cos x}$

$= \int \cos^2 x \cdot u^{100} \cdot du$ Goal: replace all "x-terms" by "u-terms".

$= \int (1-u^2) \cdot u^{100} \cdot du$ Hint: Trig-Identity: $\boxed{\sin^2 x + \cos^2 x = 1}$ Caution: NOT in the exam formula sheet

$\cos^2 x = 1 - \sin^2 x = 1 - u^2$

$= \int u^{100} - u^{102} \cdot du$ Hint: $\int x^n\, dx = \frac{1}{n+1} \cdot x^{n+1} + C$, $n \neq -1$.

$= \frac{1}{101} u^{101} - \frac{1}{103} u^{103} + C$

$= \boxed{\frac{1}{101} \sin^{101} x - \frac{1}{103} \sin^{103} x + C}$

Remark: If both sin & cos have odd powers, then SUBSTITUTE the one with HIGHER power.

★ e.g. 2. $\int \sin^5(2t) \cdot \cos^9(2t) \cdot dt$ Hint: cos part has higher power 9.

$\boxed{u = \cos(2t),\quad du = -2\sin 2t \cdot dt}$

solve for $dt = \frac{du}{-2\sin 2t}$

$= \int \sin^5(2t) \cdot u^9 \cdot \frac{du}{-2\sin 2t}$

$= \int \frac{1}{-2} \sin^4(2t) \cdot u^9\, du$ Replace $\sin^4$ by $u = \cos$ via Trig-Id

$\sin^2(2t) + \cos^2(2t) = 1$

$= \frac{1}{-2} \int (1-u^2)^2 \cdot u^9 \cdot du$ $\Rightarrow \sin^2(2t) = 1 - \cos^2(2t) = 1 - u^2$

$= \frac{1}{-2} \int (1 - 2u^2 + u^4) \cdot u^9\, du$ $\Rightarrow \sin^4(2t) = [\sin^2(2t)]^2 = (1-u^2)^2$

$= \frac{1}{-2} \int u^9 - 2 \cdot u^{11} + u^{13}\, du$

$= \frac{1}{-2} \cdot \left(\frac{1}{10} u^{10} - 2 \cdot \frac{1}{12} u^{12} + \frac{1}{14} u^{14} \right) + C$

$= \frac{1}{-2} \cdot \left(\frac{1}{10} \cdot \cos^{10}(2t) - \frac{1}{6} \cos^{12}(2t) + \frac{1}{14} \cos^{14}(2t) \right) + C$

$= \boxed{-\frac{1}{20} \cos^{10}(2t) + \frac{1}{12} \cos^{12}(2t) - \frac{1}{28} \cos^{14}(2t) + C}$

② sin · cos product without ODD power: Double Angle Formulas in the formula sheet.

$$\sin^2 x = \frac{1}{2}(1-\cos 2x), \quad \cos^2 x = \frac{1}{2}(1+\cos 2x), \quad \sin 2x = 2\sin x \cdot \cos x.$$

e.g.3. $\int \sin^2\theta \cdot \cos^2\theta \, d\theta$ Hint: D.A.F. for $\sin^2$ and $\cos^2$

$$= \int \frac{1-\cos 2\theta}{2} \cdot \frac{1+\cos 2\theta}{2} \, d\theta$$

$$= \int \frac{1}{4} \cdot (1-\cos^2 2\theta) \, d\theta \quad \text{Hint: D.A.F. for } \cos^2$$

$$= \int \frac{1}{4} \cdot \left[1 - \frac{1}{2}(1+\cos 2\cdot 2\theta)\right] \cdot d\theta \quad \text{Caution: } 2\theta \xrightarrow{\text{double}} \boxed{4\theta}$$

$$= \int \frac{1}{4} \cdot \left[\frac{1}{2} - \frac{1}{2}\cos 4\theta\right] \cdot d\theta$$

$$= \int \frac{1}{8} - \frac{1}{8}\cos 4\theta \, d\theta = \boxed{\frac{1}{8}\theta - \frac{1}{8} \cdot \frac{1}{4}\sin 4\theta + C}$$

Hint: $\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b)$

• Trig-Identities/Formulas NOT in the exam formula sheet.

$$\sin^2 x + \cos^2 x = 1, \quad \sec^2 x = \tan^2 x + 1, \quad \tan x = \frac{\sin x}{\cos x}, \quad \sec x = \frac{1}{\cos x}, \quad \cot = \frac{\cos x}{\sin x}$$

$$(\sin x)' = \cos x, \quad (\cos x)' = -\sin x, \quad (\tan x)' = \sec^2 x, \quad (\sec x)' = \tan x \cdot \sec x, \quad (\cot x)' = -\csc^2 x.$$

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③. If it is not ①/②, try to use above Trig-Id to re-write the integral.

e.g.4. $\int_{\pi/6}^{\pi/2} \frac{d\theta}{\sin^2\theta}$ Caution: Although it is Even Power, DO NOT use D.A.F.

Hint: $\frac{1}{\sin\theta} = \csc\theta$ and $(\cot\theta)' = -\csc^2\theta$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \csc^2\theta \, d\theta = -\cot\theta \Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}} = -\left[\cot\frac{\pi}{2} - \cot\frac{\pi}{6}\right]$$

$$= -[0-\sqrt{3}] = \boxed{\sqrt{3}}$$

Hint: $\cot\frac{\pi}{2} = \frac{\cos\frac{\pi}{2}}{\sin\frac{\pi}{2}} = \frac{0}{1} = 0$

$\cot\frac{\pi}{6} = \frac{\cos\frac{\pi}{6}}{\sin\frac{\pi}{6}} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

e.g.5. $\int \tan^2 x \cdot \cos^3 x \cdot dx$ Hint: $\tan x = \frac{\sin x}{\cos x} \Rightarrow \tan^2 x \cdot \cos^3 x = \frac{\sin^2 x}{\cos^2 x} \cdot \cos^3 x = \sin^2 x \cdot \cos x$

$$= \int \sin^2 x \cdot \underbrace{\cos x \, dx}$$

Hint: cos ODD power, substitute sin. $u = \sin x$

$du = \cos x \, dx$

$$= \int u^2 \cdot du = \frac{1}{3}u^3 + C = \boxed{\frac{1}{3}\sin^3 x + C}$$

Hints for some Webwork:

ww5: Rewrite the integral via DAF: $\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$, $1-\cos x = 2\sin^2\frac{x}{2}$.

ww9: Rewrite the integral via Produce-To-Sum identities (in the exam formula sheet).

$\sin A\cdot\sin B = \frac{1}{2}[\cos(A-B)-\cos(A+B)]$, $\sin A\cdot\cos B = \frac{1}{2}[\sin(A-B)+\sin(A+B)]$, $\cos A\cos B = \frac{1}{2}[\cos(A-B)+\cos(A+B)]$

tan-Trick: (ww7, 6, 8)

☆☆☆ e.g. 6. $\int \tan^3 x\cdot dx$ u-sub: $u=\sec x$, $du = \tan x\cdot\sec x\cdot dx \Rightarrow \frac{du}{\tan x\sec x} = dx$

$= \int \tan^3 x\cdot\frac{1}{\tan x\sec x}du = \int \frac{\tan^2 x}{\sec x}du$, Hint: $\tan^2 x = \sec^2 x - 1 = u^2-1$

$\overset{u=\sec x}{=\!=} \int \frac{u^2-1}{u}du$

$= \int u - \frac{1}{u}du = \frac{1}{2}u^2 - \ln|u| + C$

$= \boxed{\frac{1}{2}\sec^2 x - \ln|\sec x| + C}$

☆☆☆ eg 7. $\int \tan^4 x\cdot dx$ $u=\tan x$, $du = \sec^2 x\,dx = (\tan^2 x+1)\cdot dx = (u^2+1)\cdot dx$

$\frac{du}{u^2+1} = dx$

$= \int \frac{u^4}{u^2+1}du$ Hint: $u^4 = (u^2+1)\cdot(u^2-1)+1$

$= \int \frac{(u^2+1)(u^2-1)}{u^2+1} + \frac{1}{u^2+1}du = \int (u^2-1) + \frac{1}{u^2+1}du$

$= \frac{1}{3}u^3 - u + \tan^{-1}u + C$ Hint: $f^{-1}(f(x)) = x$

$= \frac{1}{3}\tan^3 x - \tan x + \tan^{-1}(\tan x) + C$

$= \boxed{\frac{1}{3}\tan^3 x - \tan x + x + C}$

☆☆☆ eg 8. If $\int \sec^3 x\,dx = \frac{1}{2}(\sec x\cdot\tan x + \ln|\sec x+\tan x|) + C$, find $\int \sec^3(ax)dx$.

Hint: u-sub, $u = ax$, $du = a\cdot dx$.

$\int \sec^3(ax)dx = \int \sec^3(u)\frac{du}{a} = \frac{1}{a}\left[\frac{1}{2}(\sec u\cdot\tan u + \ln|\sec u+\tan u|)\right] + C$

$= \boxed{\frac{1}{2a}\cdot(\sec(ax)\cdot\tan(ax) + \ln|\sec(ax)+\tan(ax)|) + C}$

§7.3 Trignometric Substitution

Trig-Sub Rules:

☆ ① $\sqrt{a^2-b^2x^2}$, $bx = a\cdot\sin\theta$

② $\sqrt{b^2x^2-a^2}$, $bx = a\cdot\sec\theta$

③ $\sqrt{b^2x^2+a^2}$, $bx = a\cdot\tan\theta$

$\Longrightarrow$ change the integral in terms of x. into a trig-integral in §7.2.

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eg 1 (S1b) $\displaystyle\int \frac{8\,dx}{x^2\sqrt{16-x^2}}$.

Step 1: Identify the type and find a, b.

type ①, $\sqrt{a^2-b^2x^2}$, $a=4$, $b=1$.

Step 2: Choose the corresponding Trig-Sub with a, b.

$x = 4\sin\theta$, $dx = 4\cos\theta\cdot d\theta$

Step 3: SUBSTITUTE x by θ.

Step 4: Remove square root via Trig-Identity: $1-\sin^2\theta = \cos^2\theta$.

$\sqrt{16-(4\sin\theta)^2} = \sqrt{16-16\sin^2\theta} = \sqrt{16(1-\sin^2\theta)} = \sqrt{4^2\cdot\cos^2\theta} = 4\cos\theta$.

Step 5: Simplify. Hint: $\frac{1}{\sin^2\theta} = \csc^2\theta$.

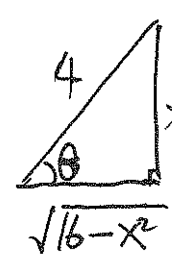
Step 6: Evaluate the Trig-Integral via method of §7.2.

Hint: $(\cot\theta)' = -\csc^2\theta$.

$\boxed{\text{Step 7:}}$ (Solve Triangle, $\theta \longrightarrow x$)

$x = 4\sin\theta \Rightarrow \sin\theta = \frac{x}{4}$

$\Rightarrow \cot\theta = \frac{\text{adj}}{\text{opp}} = \frac{\sqrt{16-x^2}}{x}$



$$\overset{\text{step3}}{=} \int \frac{8\cdot 4\cos\theta\cdot d\theta}{(4\sin\theta)^2\cdot\sqrt{16-(4\sin\theta)^2}}$$

$$\overset{\text{step4}}{=} \int \frac{8\cdot 4\cos\theta\cdot d\theta}{16\sin^2\theta\cdot 4\cos\theta}$$

$$\overset{s5}{=} \int \frac{1}{2}\cdot\frac{1}{\sin^2\theta}\,d\theta$$

$$\overset{s6}{=} \int \frac{1}{2}\cdot\csc^2\theta\,d\theta$$

$$= \frac{1}{2}\cdot(-\cot\theta) + C$$

$$\overset{s7}{=} \boxed{\frac{1}{2}\left(-\frac{\sqrt{16-x^2}}{x}\right) + C}$$

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eg 2. $\int x\cdot\sqrt{3-x^2}\,dx$. u-Sub: $u = 3-x^2$, $du = -2x\,dx$,

$$= \int \sqrt{u}\,\frac{du}{-2} = \frac{1}{-2}\cdot\frac{1}{1+\frac{1}{2}}\cdot u^{1+\frac{1}{2}} + C = -\frac{1}{3}u^{\frac{3}{2}} + C = \boxed{-\frac{1}{3}(3-x^2)^{\frac{3}{2}} + C}$$

Remark: eg 2 can also be evaluated via Trig-Sub of type ①: $x = \sqrt{3}\cdot\sin\theta$, which is slower.

$\sqrt{3-x^2} = \sqrt{(\sqrt{3})^2 - x^2}$

eg 3. $\int \frac{dx}{\sqrt{9x^2-1}}$ type ②: $\sqrt{9x^2-1} = \sqrt{(3x)^2 - 1^2}$ $b=3,\ a=1$.

$\uparrow b \qquad \uparrow a$.

Trig-Sub: $\boxed{3x = \sec\theta}$ $\quad = \sqrt{\sec^2\theta - 1} = \sqrt{\tan^2\theta} = \tan\theta$.

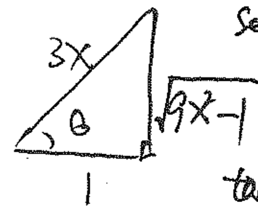
$\int \frac{1}{\sqrt{(3x)^2-1}}\cdot dx$

$x = \frac{1}{3}\sec\theta$

$= \int \frac{1}{\sqrt{\sec^2\theta - 1}} \cdot \frac{1}{3}\sec\theta\cdot\tan\theta\, d\theta$ $\qquad dx = \frac{1}{3}\sec\theta\cdot\tan\theta\cdot d\theta$.

$= \int \frac{1}{\cancel{\tan\theta}}\cdot\frac{1}{3}\sec\theta\cdot\cancel{\tan\theta}\, d\theta$

$= \frac{1}{3}\cdot\ln|\sec\theta + \tan\theta| + C$. Solve Triangle:



$\sec\theta = 3x = \frac{3x}{1}$

$\tan\theta = \frac{\sqrt{9x^2-1}}{1}$

$= \boxed{\frac{1}{3}\cdot\ln\left|3x + \sqrt{9x^2-1}\right| + C}$

eg 4. $\int \frac{dx}{(4x^2+25)^{\frac{3}{2}}}$ Hint: $(4x^2+25)^{\frac{3}{2}} = \left(\sqrt{4x^2+25}\right)^3$. type ③

$\left(\begin{array}{l} b=2,\ a=5 \\ 2x = 5\tan\theta \end{array}\right)$

$x = \frac{5}{2}\cdot\tan\theta$

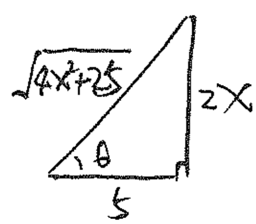
$= \int \frac{1}{(5\sec\theta)^3}\cdot\frac{5}{2}\cdot\sec^2\theta\, d\theta$ $\qquad dx = \frac{5}{2}\sec^2\theta\cdot d\theta$ $\qquad = \left(\sqrt{25\tan^2\theta + 25}\right)^3$

$= \left(\sqrt{25(\tan^2\theta+1)}\right)^3 = \left(\sqrt{25\cdot\sec^2\theta}\right)^3$

$= (5\cdot\sec\theta)^3$

$= \int \frac{1}{50}\cdot\frac{1}{\sec\theta}\, d\theta$ $\qquad$ Hint: $\sec\theta = \frac{1}{\cos\theta}$

$= \int \frac{1}{50}\cdot\cos\theta\cdot d\theta$



$\tan\theta = \frac{2x}{5}$

$= \frac{1}{50}\cdot\sin\theta + C$ $\qquad$ Solve Trig.

$\sin\theta = \frac{2x}{\sqrt{4x^2+25}}$

$= \boxed{\frac{1}{50}\cdot\frac{2x}{\sqrt{4x^2+25}} + C}$

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ww Hints:

ww4: In order to evaluate trig-integral $\int \tan^3\theta\cdot\sec\theta\, d\theta$, try substitution $\boxed{u = \sec\theta}$

ww7: Use 'complete the square' trick first.

$\sqrt{(x-c)^2 - a^2}$ is similar to $\sqrt{x^2 - a^2}$, the trig-sub is $\boxed{x - c = a\sec\theta}$